DAY EIGHT

Rotational Motion

Learning & Revision for the Day

- Concept of Rotational Motion
- Equation of Rotational Motion
- Moment of Force or Torque
- Angular Momentum
- Law of Conservation of Angular Momentum
- Equilibrium of a Rigid Bodies
- Rigid Body Rotation

Concept of Rotational Motion

In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

- Rotational motion is characterised by angular displacement $d\theta$ and angular velocity $\omega = \frac{d\theta}{dt}$.
- If angular velocity is not uniform, then rate of change of angular velocity is called the **angular acceleration**.

  Angular acceleration, $\alpha = \frac{d\omega}{dt}$.

  SI unit of angular acceleration is rad/s^2.
- Angular acceleration α and linear tangential acceleration $\mathbf{a}_t$ are correlated as $\mathbf{a}_t = \alpha \times \mathbf{r}$.

Equation of Rotational Motion

If angular acceleration α is uniform, then equations of rotational motion may be written as

(i) $\omega = \omega_0 + \alpha t$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 - \omega_0^2 = 2\alpha\theta$

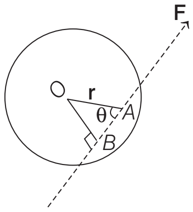
(iv) $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n-1)$

Moment of Force or Torque

Torque (or moment of a force) is the turning effect of a force applied at a point on a rigid body about the axis of rotation.

Mathematically, torque, $\tau = \mathbf{r} \times \mathbf{F} = |\mathbf{r} \times \mathbf{F}|\,\hat{\mathbf{n}} = r\,F \sin\theta\,\hat{\mathbf{n}}$

where, $\hat{\mathbf{n}}$ is a unit vector along the axis of rotation. Torque is an axial vector and its SI unit is newton-metre (N-m).



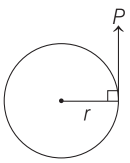
- The torque about axis of rotation is independent of choice of origin O, so long as it is chosen on the axis of rotation AB.
- Only normal component of force contributes towards the torque. Radial component of force does not contribute towards the torque.
- A torque produces angular acceleration in a rotating body. Thus, torque, $\tau = I\alpha$
- Moment of a couple (or torque) is given by product of position vector **r** between the two forces and either force **F**. Thus, $\tau = \mathbf{r} \times \mathbf{F}$
- If under the influence of an external torque, τ the given body rotates by $d\theta$, then work done, $dW = \tau \cdot d\theta$.
- In rotational motion, power may be defined as the scalar product of torque and angular velocity, i.e. Power $P = \tau \cdot \omega$.

Angular Momentum

The moment of linear momentum of a given body about an axis of rotation is called as its angular momentum. If **p** be the linear momentum of a particle and **r** is its position vector from the point of rotation, then

Angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p} = r\,p \sin\theta\,\hat{\mathbf{n}}$

where, $\hat{\mathbf{n}}$ is a unit vector in the direction of rotation. Angular momentum is an axial vector and its SI unit is kg-m^{2}s^{-1} or J-s.



- For rotational motion of a rigid body, angular momentum is equal to the product of angular velocity ω and moment of inertia of the body I about the axis of rotation.
  Mathematically, $L = I\omega$.
- According to the second law of rotational motion, the rate of change of angular momentum of a body is equal to the external torque τ applied on it and takes place in the direction of torque. Thus,
  $$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\,\frac{d\omega}{dt} = I\alpha \qquad \left[\because \alpha = \frac{d\omega}{dt}\right]$$
- Total effect of a torque applied on a rotating body in a given time is called angular impulse. Angular impulse is equal to total change in angular momentum of the system in given time. Thus, angular impulse,
  $$J = \int_0^{\Delta t} \tau\,dt = \Delta L = L_f - L_i$$
- The angular momentum of a system of particles about the origin is $\quad L = \sum^{n} r_i \times p_i$

Law of Conservation of Angular Momentum

According to the law of conservation of angular momentum, if no external torque is acting on a system, then total vector sum of angular momentum of different particles of the system remains constant.

We know that, $\quad \dfrac{dL}{dt} = \tau_{\text{ext}}$

Hence, if $\tau_{\text{ext}} = 0$, then $\dfrac{dL}{dt} = 0 \Rightarrow L =$ constant

Therefore, in the absence of any external torque, total angular momentum of a system must remain conserved.

Comparison of Linear and Rotational Motion

	Linear Motion	Rotational Motion
1.	Linear momentum, $p = mv$	Angular momentum, $L = I\omega$, $L = \sqrt{2IE}$
2.	Force, $F = ma$	Torque, $T = I\alpha$
3.	Kinetic energy, $E = \frac{1}{2}mv^2$	Rotational energy, $E = \frac{1}{2}I\omega^2$

Equilibrium of a Rigid Bodies

For mechanical equilibrium of a rigid body, two condition need to be satisfied.

1. Translational Equilibrium

A rigid body is said to be in translational equilibrium, if it remains at rest or moving with a constant velocity in a particular direction. For this, the net external force or the vector sum of all the external form acting on the body must be zero,

i.e. $\quad \mathbf{F} = 0$ or $F = \Sigma F_i = 0$

2. Rotational Equilibrium

A rigid body is said to be in rotational equilibrium, if the body does not rotate or rotates with constant angular velocity. For this, the net external torque or the vector sum of all the torques acting on the body is zero.

For the body to be in rotational equilibrium,

$$\tau_{\text{ext}} = 0, \frac{dL}{dt} = 0 \text{ or } L = \text{constant}$$

Rigid Body Rotation

Spinning

When the body rotates in such a manner that its axis of rotation does not move, then its motion is called spinning motion.

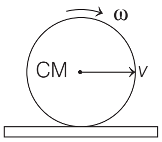
In spinning rotational kinetic energy is given by, $K_R = \dfrac{1}{2}I\omega^2$.

Rotational kinetic energy is a scalar having SI unit joule (J). Rotational kinetic energy is related to angular momentum as per relation,

$$K_R = \frac{L^2}{2I} \text{ or } L = \sqrt{2\,IK_R}$$

Pure Rolling Motion

Let a rigid body, having symmetric surface about its centre of mass, is being spined at a certain angular speed and placed on a surface, so that plane of rotation is perpendicular to the surface. If the body is simultaneously given a translational motion too, then the net motion is called **rolling motion.**



The total kinetic energy in rolling motion,

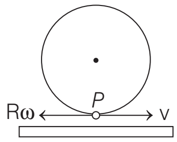
$$K = K_T + K_R$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2\left(\frac{v^2}{R^2}\right) \qquad [\because v = r\omega]$$

$$K = \frac{1}{2}mv^2\left(1 + \frac{v^2}{R^2}\right)$$

Rolling Without Slipping

If the given body rolls over a surface such that there is no relative motion between the body and the surface at the point of contact, then the motion is called **rolling without slipping**.

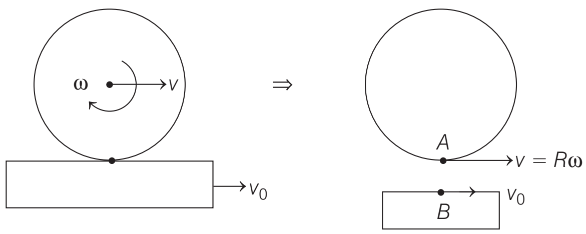


Impure Rolling Motion

In impure rolling motion, the point of contact of the body with the platform is not relatively at rest w.r.t. platform on which, it is performing rolling motion, as a result sliding occurs at point of contact.

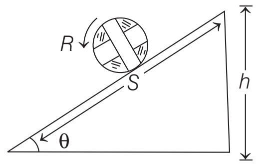
For impure rolling motion, $v_{AB} \neq 0$ i.e. $v - R\omega \neq v_0$

If platform is stationary, i.e. $v_0 = 0$, then $v \neq R\omega$



Rolling on an Inclined Plane

When a body of mass m and radius R rolls down on inclined plane of height h and angle of inclination θ, it loses potential energy. However, it acquires both linear and angular speeds and hence gain kinetic energy of translation and that of rotation.



By conservation of mechanical energy, $mgh = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

- **Velocity at the lowest point** $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$
- **Acceleration in motion** From second equation of motion, $v^2 = u^2 + 2as$

  By substituting $u = 0$, $s = \dfrac{h}{\sin\theta}$ and $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$, we get

  $$a = \frac{g\sin\theta}{1 + \dfrac{K^2}{R^2}}$$

- **Time of descent** From first equation of motion, $v = u + at$

  By substituting $u = 0$ and value of v and a from above expressions $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left[1 + \dfrac{K^2}{R^2}\right]}$

  From the above expressions, it is clear that,

  $$v \propto \frac{1}{\sqrt{1 + \dfrac{K^2}{R^2}}}; a \propto \frac{1}{1 + \dfrac{K^2}{R^2}}; t \propto \sqrt{1 + \frac{K^2}{R^2}}$$

Important Terms Related to Inclined Plane

- Here, factor $\left(\dfrac{K^2}{R^2}\right)$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.
- Velocity, acceleration and time of descent (for a given inclined plane) all depends on $\dfrac{K^2}{R^2}$. Lesser the moment of inertia of the rolling body lesser will be the value of $\dfrac{K^2}{R^2}$. So, greater will be its velocity and acceleration and lesser will be the time of descent.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 When a ceiling fan is switched on, it makes 10 revolutions in the first 3s. Assuming a uniform angular acceleration, how many rotation it will make in the next 3s?

(a) 10 (b) 20 (c) 30 (d) 40

2 The drive shaft of an automobile rotates at 3600 rpm and transmits 80 HP up from the engine to the rear wheels. The torque developed by the engine is

(a) 16.58 Nm (b) 0.022 Nm
(c) 158.31 Nm (d) 141.6 Nm

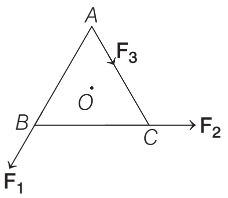
3 The moment of the force, $\mathbf{F} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ at $(2, 0, -3)$, about the point $(2, -2, -2)$, is given by ➔ NEET 2018

(a) $-7\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ (b) $-4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 8\hat{\mathbf{k}}$
(c) $-8\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$ (d) $-7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

4 An automobile moves on a road with a speed of $54\ \text{kmh}^{-1}$. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is $3\ \text{kg m}^2$. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to the wheel is ➔ CBSE AIPMT 2015

(a) $6.66\ \text{kg m}^2\text{s}^{-2}$ (b) $8.58\ \text{kg m}^2\text{s}^{-2}$
(c) $10.86\ \text{kg m}^2\text{s}^{-2}$ (d) $2.86\ \text{kg m}^2\text{s}^{-2}$

5 ABC is an equilateral triangle with O as its centre. $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ represent three forces acting along the sides AB, BC and AC, respectively. If the total torque about O is zero, then the magnitude of $\mathbf{F}_3$ is ➔ CBSE AIPMT 2012



(a) $F_1 + F_2$ (b) $F_1 - F_2$ (c) $\dfrac{F_1 + F_2}{2}$ (d) $2(F_1 + F_2)$

6 The instantaneous angular position of a point on a rotating wheel is given by the equation $Q(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at ➔ CBSE AIPMT 2011

(a) $t = 0.5$ s (b) $t = 0.25$ s (c) $t = 2$ s (d) $t = 1$ s

7 If $\mathbf{F}$ is the force acting on a particle having position vector $\mathbf{r}$ and τ be the torque of this force about the origin, then ➔ CBSE AIPMT 2009

(a) $\mathbf{r} \cdot \tau \neq 0$ and $\mathbf{F} \cdot \tau = 0$ (b) $\mathbf{r} \cdot \tau > 0$ and $\mathbf{F} \cdot \tau < 0$
(c) $\mathbf{r} \cdot \tau = 0$ and $\mathbf{F} \cdot \tau = 0$ (d) $\mathbf{r} \cdot \tau = 0$ and $\mathbf{F} \cdot \tau \neq 0$

8 A string is wound round the rim of a mounded flywheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord. Negleating friction and mass of the string. The angular acceleration of the wheel is

(a) 50 rad/s^2 (b) 25 rad/s^2 (c) 12.5 rad/s^2 (d) 6.25 rad/s^2

9 A constant torque of 31.4 N-m is exerted on a pivoted wheel. If angular acceleration of wheel is 4π rad/sec^2, then the moment of inertia of the wheel is

(a) 2.5 kg-m^2 (b) 3.5 kg-m^2
(c) 4.5 kg-m^2 (d) 5.5 kg-m^2

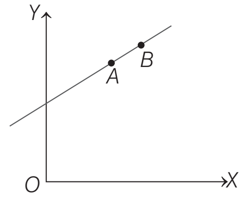
10 A force of $-F\hat{\mathbf{k}}$ acts on O_i the origin of the coordinate system. The torque about the point $(1, -l)$ is

(a) $-F(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (b) $F(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(c) $-F(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (d) $F(\hat{\mathbf{i}} - \hat{\mathbf{j}})$

11 A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?

(a) Moment of inertia (b) Angular momentum
(c) Angular velocity (d) Rotational kinetic energy

12 A particle of mass m in the XY-plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then



(a) $L_A > L_B$
(b) $L_A = L_B$
(c) the relationship between L_A and L_B depends upon the slope of the line AB
(d) $L_A < L_B$

13 In an orbital motion, the angular momentum vector is

(a) along the radius vector
(b) parallel to the linear momentum
(c) in the orbital plane
(d) perpendicular to the orbital plane

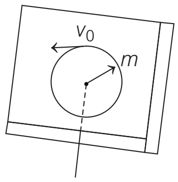
14 When a mass is rotating in a plane about a fixed point, its angular momentum is directed along ➔ CBSE AIPMT 2012

(a) a line perpendicular to the plane of rotation
(b) the line making an angle of 45° to the plane of rotation
(c) the radius
(d) the tangent to the orbit

15 A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

(a) $\dfrac{I_2\omega}{I_1 + I_2}$ (b) ω
(c) $\dfrac{I_1\omega}{I_1 + I_2}$ (d) $\dfrac{(I_1 + I_2)\omega}{I_1}$

16 A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hold in the plane as shown in figure.



The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is → CBSE AIPMT 2015

(a) $\frac{1}{4}mv_0^2$ (b) $2mv_0^2$ (c) $\frac{1}{2}mv_0^2$ (d) mv_0^2

17 A solid cylinder of mass 50 kg and radius 0.5 m is, free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolution s^{-2} is → CBSE AIPMT 2014

(a) 25 N (b) 50 N (c) 78.5 N (d) 157 N

18 A thin circular ring of mass M and radius R is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity ω. If two objects each of mass m be attached gently to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity → CBSE AIPMT 2009, 1998

(a) $\frac{\omega(M-2m)}{M+2m}$ (b) $\frac{\omega M}{M+2m}$ (c) $\frac{\omega(M+2m)}{M}$ (d) $\frac{\omega M}{M+m}$

19 Three objects, A : (a solid sphere), B : (a thin circular disk) and C : (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (W) required to bring them to rest, would satisfy the relation → NEET 2018

(a) $W_B > W_A > W_C$ (b) $W_A > W_B > W_C$
(c) $W_C > W_B > W_A$ (d) $W_A > W_C > W_B$

20 Initial angular velocity of a circular disc of mass M is ω_1. Then, two small spheres of m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

21 Two discuss of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is → NEET 2017

(a) $\frac{1}{2}I(\omega_1+\omega_2)^2$ (b) $\frac{1}{4}I(\omega_1-\omega_2)^2$
(c) $I(\omega_1-\omega_2)^2$ (d) $\frac{I}{8}(\omega_1-\omega_2)^2$

22 Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then → NEET 2016

(a) $L_A = \frac{L_B}{2}$ (b) $L_A = 2L_B$ (c) $L_B > L_A$ (d) $L_A > L_B$

23 A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with angular speed twice that of the sphere. The ratio of their kinetic energies of rotation $\left(\frac{E_{\text{Sphere}}}{E_{\text{Cylinder}}}\right)$ will be → NEET 2016

(a) 2 : 3 (b) 1 : 5 (c) 1 : 4 (d) 3 : 1

24 If the angular momentum of any rotating body increasing by 200%, then the increase in its kinetic energy

(a) 400% (b) 800% (c) 200% (d) 100%

25 A small object of uniform density rolls up a curved surface with an initial velocity v'. It reaches upto a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is → NEET 2013

(a) ring (b) solid sphere
(c) hollow sphere (d) disc

26 An inclined plane makes an angle of 30° with the horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration equal to

(a) $\frac{g}{3}$ (b) $\frac{2g}{3}$ (c) $\frac{5g}{7}$ (d) $\frac{5g}{14}$

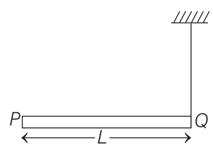
27 A circular disc rolls down an inclined plane. The ratio of rotational kinetic energy to total kinetic energy is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

28 The ratio of the acceleration for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is → CBSE-AIPMT 2014

(a) 5 : 7 (b) 2 : 3 (c) 2 : 5 (d) 7 : 5

29 A rod PQ of mass M and length L is hinged at end P. The rod is kepts horizontal by a massless string tied to point Q as shown in figure.



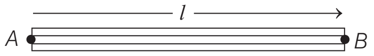
When string is cut, the initial angular acceleration of the rod is → NEET 2013

(a) $\frac{3g}{2L}$ (b) $\frac{g}{L}$
(c) $\frac{2g}{L}$ (d) $\frac{2g}{3L}$

DAY PRACTICE SESSION 2

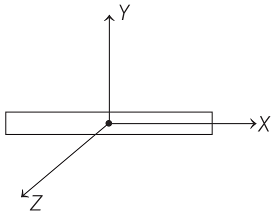
PROGRESSIVE QUESTIONS EXERCISE

1 A uniform rod of length l and mass m is free to rotate in a vertical plane about A. The rod initially in horizontal position is released. The initial angular acceleration of the rod is $\left(\text{Moment of inertia of rod about } A \text{ is } \frac{ml^2}{3}\right)$



(a) $\frac{3g}{2l}$
(b) $\frac{2l}{3g}$
(c) $\frac{3g}{2l^2}$
(d) $mg\frac{l}{2}$

2 A uniform rod of mass M and length L is free to rotate in XY plane i.e. about Y-axis. If a force of $\mathbf{F} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ N is acting on rod at (L/2, 0, 0) in the situation as shown in figure. The angular acceleration of rod is (take, $M = 6$ kg, $L = 4$ m)



(a) $-\frac{3}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{k}}$
(b) $-\frac{3}{2}\hat{\mathbf{j}}$
(c) $\frac{1}{2}\hat{\mathbf{k}}$
(d) $4\hat{\mathbf{j}}$

3 A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder, if the rope is pulled with a force of 30 N?

(a) 25 m/s^2
(b) 0.25 rad/s^2
(c) 25 rad/s^2
(d) 5 m/s^2

4 A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $M\omega^2L/2$
(b) $M\omega^2L$
(c) $M\omega^2 L/4$
(d) $M\omega^2L^2/2$

5 A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

(a) $\frac{L}{4}$
(b) $2L$
(c) $4L$
(d) $\frac{L}{2}$

6 A couple consisting of two forces F_1 and F_2 each equal to 5 N is acting at the rim of a disc of mass 2 kg and radius $\frac{1}{2}$ m for 5 s. Initially, the disc is at rest, the final angular momentum (in Nm-s) of the disc is

(a) 15
(b) 20
(c) 25
(d) 30

7 A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\left(\frac{M\omega}{M+4m}\right)$
(b) $\frac{(M+4m)\omega}{M}$
(c) $\frac{(M-4m)\omega}{M+4m}$
(d) $\left(\frac{M\omega}{4m}\right)$

8 A thin and circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. If another disc of same dimensions but of mass M/4 is placed gently on the first disc coaxially, then the new angular velocity of the system is

(a) $\frac{5}{4}\omega$
(b) $\frac{2}{3}\omega$
(c) $\frac{4}{5}\omega$
(d) $\frac{3}{2}\omega$

9 A sphere of mass m and radius r falls on a horizontal plane without slipping with the speed u. Now if it falls up vertically, the maximum height it would attain will be

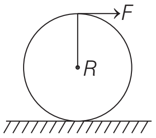
(a) $\frac{3u^2}{4g}$
(b) $\frac{5u^2}{2g}$
(c) $\frac{7u^2}{10g}$
(d) $\frac{u^2}{2g}$

10 A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere?

➜ NEET 2018

(a) Rotational kinetic energy
(b) Moment of inertia
(c) Angular velocity
(d) Angular momentum

11 A tangential force F acts at the top of a thin spherical shell of mass m and radius R. The acceleration of the shell, if it rolls without slipping is



(a) $\frac{5}{6}\frac{F}{m}$
(b) $\frac{6}{5}\frac{F}{m}$
(c) $\frac{7}{2}\frac{m}{F}$
(d) $\frac{2}{7}\frac{m}{F}$

12 A disc and a sphere of same radius but different masses role off an two inclined planes of the same altitude and length. Which ones of the objects gets to the bottom of the plane first?

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(a) Sphere
(b) Both reach all the same time
(c) Depends on their masses
(d) Disc

ANSWERS

SESSION 1	**1** (c)	**2** (c)	**3** (d)	**4** (a)	**5** (a)	**6** (d)	**7** (c)	**8** (c)	**9** (a)	**10** (b)
	11 (b)	**12** (b)	**13** (d)	**14** (a)	**15** (c)	**16** (b)	**17** (d)	**18** (b)	**19** (c)	**20** (c)
	21 (b)	**22** (c)	**23** (b)	**24** (b)	**25** (d)	**26** (d)	**27** (b)	**28** (a)	**29** (a)	
SESSION 2	**1** (a)	**2** (b)	**3** (a)	**4** (a)	**5** (a)	**6** (c)	**7** (a)	**8** (c)	**9** (c)	**10** (d)
	11 (b)	**12** (a)								

Hints and Explanations

SESSION 1

1 Angle turned in 3s, $\theta_{3s} = 2\pi \times 10 = 20\pi$ rad

From $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow 20\pi = 0 + \frac{1}{2}\alpha \times (3)^2 \Rightarrow \alpha = \frac{40\pi}{9}$ rad/s^2

Now angle turned in 6 s from the starting

$$\theta_{6s} = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times \left(\frac{40\pi}{9}\right) \times (6)^2$$

$= 80\pi$rad

$\therefore$ angle turned between $t = 3$ s to $t = 6$ s

$\theta_{\text{last}\,3\,s} = \theta_{6s} - \theta_{3s} = 80\pi - 20\pi = 60\pi$

Number of revolution

$$= \frac{60\pi}{2\pi} = 30 \text{ revolution}$$

2 From $P = \tau\omega$ and $\tau = \frac{P}{\omega}$

It is given,

$P = 80\,\text{HP} = 80 \times 746\,\text{W} = 59680\,\text{N ms}^{-1}$

$\omega = 3600$ rpm $= \frac{3600}{60} \times 2\pi$ rad s^{-1}

$= 120\,\pi$rad s^{-1}

So, $\tau = 158.31$ Nm

3 Given, $\mathbf{F} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\Rightarrow \mathbf{r}_2 = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

Moment of force $= \mathbf{r} \times \mathbf{F}$

$= (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$

$= [(2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})] \times [4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}]$

$= [0\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 1\hat{\mathbf{k}}] \times [4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}]$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & -1 \\ 4 & 5 & -6 \end{vmatrix}$$

$= \hat{\mathbf{i}}[(-6 \times 2) - (-1 \times 5)] - \hat{\mathbf{j}}[(-6 \times 0) - (-1 \times 4)] + \hat{\mathbf{k}}[(0 \times 5) - 2 \times 4]$

$= -7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

4 As velocity of an automobile vehicle,

$$v = 54 \text{ km/h} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

Angular velocity of a vehicle, $v = \omega_0 r$

$\Rightarrow \omega_0 = \frac{v}{R} = \frac{15}{0.45} = \frac{100}{3}$ rad/s

So, angular acceleration of an automobile,

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_0}{t} = \frac{0 - \frac{100}{3}}{15}$$

$= \frac{-100}{45}$ rad/s^2

Thus, average torque transmitted by its brakes to wheel

$$\tau = I\alpha \Rightarrow 3 \times \frac{100}{45} = 6.66 \text{ kg m}^2\text{s}^{-2}$$

5 If we take clockwise torque

$$\tau_{\text{net}} = \tau_{F_1} + \tau_{F_2} - \tau_{F_3}$$
$$0 = F_1 r + F_2 r - F_3 r$$
$$\Rightarrow F_3 = F_1 + F_2$$

6 According to question, torque $\tau = 0$

It means that, $\alpha = 0$

$$\alpha = \frac{d^2\theta}{dt^2}$$

Given, $\theta(t) = 2t^3 - 6t^2$

So, $\frac{d\theta}{dt} = 6t^2 - 12t \Rightarrow \alpha = \frac{d^2\theta}{dt^2} = 12t - 12$

$12t - 12 = 0 \Rightarrow t = 1$ s

7 $\tau = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r} =$ position vector

$\mathbf{F} =$ force $\Rightarrow \tau = |\mathbf{r}||\mathbf{F}|\sin\theta\,\hat{\mathbf{n}}$

Torque is perpendicular to both $\mathbf{r}$ and $\mathbf{F}$. So, dot product of two vectors will be zero.

$\therefore \tau \cdot \mathbf{r} = 0 \Rightarrow \mathbf{F} \cdot \tau = 0$

8 Here, $M = 20$ kg, $R = 20$ cm $= \frac{1}{5}$ m

Moment of inertia of flywheel about its axis is

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 20 \times \left(\frac{1}{5}\right)^2 = 0.4 \text{ kg-m}^2$$

$\tau = I\alpha$

where, α is the angular acceleration

$$\alpha = \frac{\tau}{I} = \frac{FR}{I} = \frac{25 \times \frac{1}{5}}{0.4} = \frac{5}{0.4} = 12.5 \text{ rad/s}^2$$

9 $I = \frac{\tau}{\alpha} = \frac{31.4}{4\pi} = 2.5$ kg-m^2

10 $F = -F\hat{\mathbf{k}}$ or $\mathbf{r} = (\hat{\mathbf{i}} - \hat{\mathbf{j}})$

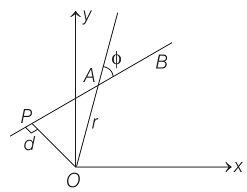
$\tau = \mathbf{r} \times \mathbf{F} = (\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (-F\hat{\mathbf{k}})$

$= -\mathbf{F}(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) + \mathbf{F}(\hat{\mathbf{j}} \times \hat{\mathbf{k}}) = -\mathbf{F}(-\hat{\mathbf{j}}) + F(\hat{\mathbf{i}})$

$= \mathbf{F}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

11 Taking the same mass of sphere, if radius is increased, then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change. So, the reaction at the other end will be equal to F_C.

12 From the definition of angular momentum,



$\mathbf{L} = \mathbf{r} \times \mathbf{p} = rmv\sin\phi(-\mathbf{k})$

Therefore, the magnitude of L is

$L = mvr\sin\phi = mvd$

where, $d = r\sin\phi$ is the distance of closest approach of the particle to the origin. As d is same for both the particles, hence $L_A = L_B$.

13 If $\mathbf{p}$ be the linear momentum of a particle and $\mathbf{r}$ is its position vector from the point of rotation, then angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p} = rp\sin\theta\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is unit vector in the direction of rotation. Hence, angular momentum vector is perpendicular to orbital plane.

14 As we know that

Angular momentum, $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

So, here angular momentum is directed along a line perpendicular to the plane of rotation.

15 The angular momentum of a disc of moment of inertia I_1 and rotating about its axis with angular velocity ω is

$$L_1 = I_1\omega$$

When a round disc of moment of inertia I_2 is placed on first disc, then angular momentum of the combination is

$L_2 = (I_1 + I_2)\omega'$

In the absence of any external torque, angular momentum remains conserved i.e. $L_1 = L_2$

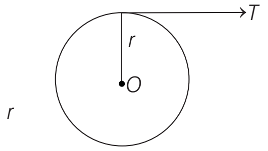
$$I_1\omega = (I_1 + I_2)\omega' \Rightarrow \omega' = \frac{I_1\omega}{I_1 + I_2}$$

16 Angular momentum remains constant because of the torque of tension is zero.

$$\Rightarrow L_i = L_f \Rightarrow mv_0R = mv\frac{R}{2} \Rightarrow v = 2v_0$$

$$\Rightarrow \text{KE}_f = \frac{1}{2}m(2v_0)^2 = 2mv_0^2$$

17 $Tr = I\alpha$



$$T = \frac{I\alpha}{r} = \frac{mr^2}{2} \times \frac{\alpha}{r} = \frac{mr\alpha}{2}$$

$$= \frac{50 \times 0.5 \times 2 \times 2\pi}{2}\text{N} = 157 \text{ N}$$

18 According to question by applying conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

In the given case

$$I_1 = MR^2 \text{ or } I_2 = MR^2 + 2mR^2$$

$$\omega_1 = \omega$$

Then, $\omega_2 = \frac{I_1}{I_2}\omega = \frac{M}{M + 2m}\omega$

19 Work done required to bring an object to rest is given as $W = \frac{1}{2}I\omega^2$

where, I is the moment of inertia and ω is the angular velocity.

Since, here all the objects spin with the same ω, this means, $W \propto I$

As, I_A (for a solid sphere) $= \frac{2}{5}MR^2$

I_B (for a thin circular disk) $= \frac{1}{2}MR^2$

I_C (for a circular ring) $= MR^2$

$$\therefore W_A : W_B : W_C = I_A : I_B : I_C$$

$$= \frac{2}{5}MR^2 : \frac{1}{2}MR^2 : MR^2 = \frac{2}{5} : \frac{1}{2} : 1$$

$$= 4 : 5 : 10$$

$$\Rightarrow W_A < W_B < W_C$$

20 Conservation of angular momentum gives

$$\frac{1}{2}MR^2\omega_1 = \left(\frac{1}{2}MR^2 + 2mR^2\right)\omega_2$$

$$\Rightarrow MR^2\omega_1 = R^2(M + 4m)\omega_2$$

$$\therefore \omega_2 = \left(\frac{M}{M + 4m}\right)\omega_1$$

21 Angular momentum before contact

$$= I_1\omega_1 + I_2\omega_2$$

Angular momentum after the discs brought into contact

$$= I_{\text{net}}\omega = (I_1 + I_2)\omega$$

So, final angular speed of system $= \omega$

$$= \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

Now, to calculate loss of energy, we subtract initial and final energies of system.

$\Rightarrow$ Loss of energy

$$= \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2 - \frac{1}{2}(2I)\omega_1\omega_2$$

$$= \frac{1}{2}I(\omega_1 - \omega_2)^2$$

22 As we know that, the kinetic energy of a rotating body,

$$\text{KE} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{L^2}{2I}$$

Also, angular momentum, $L = I\omega$

Thus, $K_A = K_B$

$$\Rightarrow \frac{1}{2}\frac{L_A^2}{I_A} = \frac{1}{2}\frac{L_B^2}{I_B} \Rightarrow \left(\frac{L_A}{L_B}\right)^2 = \frac{I_A}{I_B}$$

$$\Rightarrow \frac{L_A}{L_B} = \sqrt{\frac{I_A}{I_B}} \Rightarrow L \propto \sqrt{I}$$

$$\therefore L_A < L_B \quad [\because I_B > I_A]$$

23 KE of sphere,

$$K_S = \frac{1}{2}I\omega_1^2 = \frac{1}{2}\frac{2}{5}mR^2\omega_1^2 = \frac{1}{5}mR^2\omega_1^2$$

KE of cylinder,

$$K_C = \frac{1}{2}\frac{1}{2}mR^2\omega_2^2 = \frac{1}{4}mR^2\omega_2^2$$

$$\therefore \frac{K_S}{K_C} = \frac{\frac{mR^2\omega_1^2}{5}}{\frac{mR^2\omega_2^2}{4}} = \frac{4}{5}\frac{\omega_1^2}{\omega_2^2} = \frac{4}{5}\frac{\omega_1^2}{(2\omega_1)^2} = \frac{1}{5}$$

(given, $\omega_2 = 2\omega_1$)

24
$$E = \frac{L^2}{2I} \text{ or } E \propto L^2 \text{ or } \frac{E_2}{E_1} = \left(\frac{L_2}{L_1}\right)^2$$

$\because L_2 = 200\%$ of $(L_1) + L_1$

$$= 2L_1 + L_1 = 3L_1$$

$$\therefore \frac{E_2}{E_1} = \left(\frac{3L_1}{L_1}\right)^2 \text{ or } E_2 = 9E_1$$

Increment in kinetic energy

$$\Delta E = E_2 - E_1 = 9E_1 - E_1$$

$$\Delta E = 8E_1$$

$\therefore \frac{\Delta E}{E_1} = 8$ or percentage increase $= 800\%$

25 As, $v = \sqrt{\frac{2gh}{1 + \frac{K^2}{r^2}}}$

Given, $h = \frac{3v^2}{4g}$

$$v^2 = \frac{2gh}{1 + \frac{K^2}{r^2}} = \frac{2g(3v^2)}{4g\left(1 + \frac{K^2}{r^2}\right)} = \frac{6gv^2}{4g\left(1 + \frac{K^2}{r^2}\right)}$$

$$1 = \frac{3}{2\left(1 + \frac{K^2}{r^2}\right)} \Rightarrow \frac{K^2}{r^2} = \frac{1}{2} \Rightarrow K^2 = \frac{1}{2}r^2$$

[equation of disc]

Hence, object is disc.

26 $a = \dfrac{g\sin\theta}{1 + \frac{k^2}{R^2}} = \dfrac{g\sin\theta}{1 + \frac{2}{5}} = \dfrac{g/2}{7/5} = \dfrac{5g}{14}$

$\left[\text{as}, \theta = 30° \text{ and } \frac{K^2}{R^2} = \frac{2}{5}\right]$

27 Rotational kinetic energy, $K_R = \frac{1}{2}I\omega^2$

$$K_R = \frac{1}{2} \times \frac{MR^2}{2} \times \omega^2 = \frac{1}{4}mv^2$$

Translational kinetic energy,

$K_T = \frac{1}{2}mv^2$

Total kinetic energy, $= K_T + K_R$

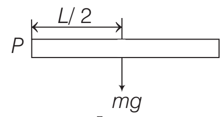
$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

$$\frac{\text{Rotational Kinetic energy}}{\text{Total Kinetic energy}} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \frac{1}{3}$$

28 $a_{\text{slipping}} = g\sin\theta$

$$a_{\text{rolling}} = \frac{g\sin\theta}{1 + \frac{K^2}{r^2}} = \frac{5}{7}g\sin\theta \Rightarrow \frac{a_{\text{rolling}}}{a_{\text{slipping}}} = \frac{5}{7}$$

29 Torque on the rod = Moment of weight of the rod about P



$$\tau = mg\frac{L}{2} \quad \text{...(i)}$$

$\therefore$ Moment of Inertia of rod about

$$P = \frac{ML^2}{3} \quad \text{...(ii)}$$

As $\tau = I\alpha$

From Eqs. (i) and (ii), we get

$$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$$

$$\therefore \alpha = \frac{3g}{2L}$$

SESSION 2

1 The moment of inertia of the uniform rod about an axis through one end and perpendicular to length is

$$I = \frac{ml^2}{3}$$

where, m is mass of rod and l its length.

Torque $(\tau = I\alpha)$ acting on centre of gravity of rod is given by

$$\tau = mg\frac{l}{2} \text{ or } I\alpha = mg\frac{l}{2} \text{ or } \frac{ml^2}{3}\alpha = mg\frac{l}{2}$$

$$\therefore \quad \alpha = \frac{3g}{2l}$$

2 $I = \frac{ML^2}{12} = \frac{6 \times 4 \times 4}{12} = 8 \text{ kg m}^{-2}$

From $\tau = \mathbf{r} \times \mathbf{F} = [2\hat{\mathbf{i}} \times (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})]$

$$= 4\hat{\mathbf{k}} - 12\hat{\mathbf{j}}$$

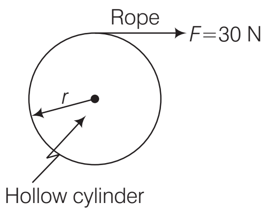
As the rod can rotate along Y-axis, so

$$\tau_y = I\alpha \Rightarrow -12\hat{\mathbf{j}} = 8\alpha$$

$$\Rightarrow \quad \alpha = -\frac{3}{2}\hat{\mathbf{j}} \text{ rad s}^{-2}$$

3 Torque (τ) acting on a body and angular acceleration (α) produced in it are related as $\tau = I\alpha$.

Consider a hollow cylinder, around which a rope is wounded as shown in the figure.



Torque acting on the cylinder due to the force F is $\tau = Fr$

Now, we have $\tau = I\alpha$

where, I = moment of inertia of the cylinder about the axis through the centre = mr^2

α = angular acceleration

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}}$$

$$= \frac{100}{4} = 25 \text{ rad / s}^2$$

4 The centre of mass is at $\frac{L}{2}$ distance from the axis.

Hence, centripetal force, $F_C = M\left(\frac{L}{2}\right)\omega^2$

5 Angular momentum,

$$L = I\omega \quad \ldots\text{(i)}$$

Kinetic energy, $\text{KE} = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$ [from Eq. (i)]

$$\therefore \quad L = \frac{2\text{ KE}}{\omega}$$

Now, $L' = \frac{2\left(\frac{\text{KE}}{2}\right)}{2\omega} \Rightarrow L' = \frac{L}{4}$

6 Torque acting on disc is,

$$\tau = F_1 r + F_2 r = 5 \times \frac{1}{2} + 5 \times \frac{1}{2}$$

$$= 5 \text{ Nm}$$

From angular impulse-momentum theorem,

$$\int_{t_i}^{t_f} \tau\, dt = L_f - L_i$$

$$\Rightarrow \quad 5 \times 5 = L_f - 0 \Rightarrow L_f = 25 \text{ Nms}$$

7 Initial angular momentum of ring,

$L = I\omega = Mr^2\omega$

Final angular momentum of ring and four particles system

$$L = (Mr^2 + 4mr^2)\omega'$$

As there is no torque on the system, therefore angular momentum remains constant.

$$Mr^2\omega = (Mr^2 + 4mr^2)\omega'$$

$$\Rightarrow \quad \omega' = \frac{M\omega}{M + 4m}$$

8 According to conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2 = \frac{1}{2}MR^2\omega$$

$$= \left(\frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2\right)\omega_2$$

$$\therefore \quad \omega_2 = \frac{4}{5}\omega$$

9 The rolling sphere has rotational as well as translational kinetic energy.

$$\therefore \text{Kinetic energy} = \frac{I}{2}mu^2 + \frac{1}{2}I\omega^2$$

$$= \frac{I}{2}mu^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$$

$$= \frac{1}{2}mu^2 + \frac{mu^2}{5} = \frac{7}{10}mu^2$$

$\therefore$ Potential energy = kinetic energy

$$mgh = \frac{7}{10}mu^2 \text{ or } h = \frac{7u^2}{10g}$$

10 Moment of inertia of a rotating solid sphere about its symmetrical (diametric) axis is given as, $I = \frac{2}{5}mR^2$

Rotational kinetic energy of solid sphere is $K_r = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}mR^2\omega^2 = \frac{1}{5}mR^2\omega^2$

Angular velocity, $\omega = V_{\text{cm}}R$

As, we know that external torque,

$$\tau_{\text{ext}} = \frac{dL}{dt}$$

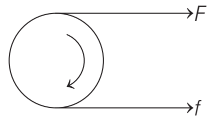
where, L is the angular momentum.

Since, in the given condition, $\tau_{\text{ext}} = 0$

$$\Rightarrow \quad \frac{dL}{dt} = 0 \text{ or } L = \text{constant}$$

Hence, when the radius of the sphere is increased keeping its mass same, only the angular momentum remains constant. But other quantities like moment of inertia, rotational kinetic energy and angular velocity changes.

11 Let f be force of friction between the shell and horizontal surface.



For translational motion,

$$F + f = ma \quad \ldots\text{(i)}$$

For rotational motion,

$$FR - fR = I\alpha = I\frac{a}{R}$$

[$a = R\alpha$ for pure rolling]

$$\Rightarrow \quad F - f = I\frac{a}{R^2} \quad \ldots\text{(ii)}$$

On, adding Eqs. (i) and (ii), we get

$$2F = \left(m + \frac{I}{R^2}\right)a = \left(m + \frac{2}{3}m\right)a = \frac{5}{3}ma$$

$$\Rightarrow \quad F = \frac{5}{6}ma \quad \left[\because I_{\text{shell}} = \frac{2}{3}mR^2\right]$$

$$\Rightarrow \quad a = \frac{6F}{5m}$$

12 Acceleration of an object rolling down an inclined plane is given by

$$a = \frac{g \sin\theta}{1 + I/mr^2}$$

where, θ = angle of inclination of the inclined plane

m = mass of the object

I = moment of inertia about the axis through centre of mass

For disc, $\frac{I}{mr^2} = \frac{1/2\, mr^2}{mr^2} = \frac{1}{2}$

For solid sphere,

$$\frac{I}{mr^2} = \frac{2/5mr^2}{mr^2} = \frac{2}{5}$$

For hollow sphere,

$$\frac{I}{mr^2} = \frac{2/3mr^2}{mr^2} = \frac{2}{3}$$

$$\therefore \quad a_{\text{disc}} = \frac{g \sin\theta}{1 + \frac{1}{2}} = \frac{2}{3}g \sin\theta$$

$$= 0.66\, g \sin\theta$$

$$a_{\text{solid sphere}} = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{5}{7}g \sin\theta$$

$$= 0.71\, g \sin\theta$$

$$a_{\text{hollow sphere}} = \frac{g \sin\theta}{1 + \frac{2}{3}} = \frac{3}{5}g \sin\theta$$

$$= 0.6\, g \sin\theta$$

Clearly, $a_{\text{solid sphere}} > a_{\text{disk}} > a_{\text{hollow sphere}}$

Type of sphere is not mentioned in the question. Therefore, we will assume the given sphere as solid sphere.

$\therefore \quad a_{\text{solid sphere}} = a_{\text{hollow sphere}} > a_{\text{disk}}$